Opportunistic Organized Crime Groups: Theory and Evidence from the Merseyside, U.K.

Supplementary materials

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A Proofs

Lemma 1

Proof. A strategy $s = (s_1, s_2, \ldots, s_M)$ is a permutation of the integers $(1, 2, \ldots, M)$ which specifies the order in which the OCG will search for a free area. If the first area which is free is the k^{th} one, then the payoff associated to strategy s is $u_{s_k} - (k-1)c$, where c is the cost which the OCG pays every time he/she aims at an occupied area. The value of k depends on the concentration level of the areas, i.e. on the strategies played by other OCGs.

Consider the strategies $s_{\ldots m \to n \ldots}$ and $s_{\ldots n \to m \ldots}$ which differ only because of the order in which two consecutive areas, in position k and $k+1$, are visited. In order to rank these two strategies, we need only consider all events where all areas s_ℓ for $\ell < k$ are occupied. Indeed, if any of these areas is free strategies $s_{\ldots m \to n \ldots}$ and $s_{\ldots n \to m \ldots}$ yield the same payoff.

The condition under which strategy $s_{\ldots m \to n \ldots}$ is better than $s_{\ldots n \to m \ldots}$ can be written as

$$
\begin{aligned}
q_i^m u_m + (1-q_i^m)\left[-c + q_i^n u_n + (1-q_i^n)(-c+\Pi)\right] \quad &\geq \\
q_i^n u_n + (1-q_i^n)\left[-c + q_i^m u_m + (1-q_i^m)(-c+\Pi)\right] \quad &,
\end{aligned} \tag{1}
$$

where Π is the expected payoff of the continuation in search if both m and n are occupied.

Given that q_i^m and q_i^n are strictly positive, from (1) we obtain (omitting index i)

$$
\begin{aligned}
q^m u_m + (1-q^m)\left[-c + q^n u_n\right] &\geq q^n u_n + (1-q^n)\left[-c + q^m u_m\right] \\
(1-q^n)c + q^m q^n u_m &\geq (1-q^m)c + q^m q^n u_n \\
u_m - \frac{1}{q^m}c &\geq u_n - \frac{1}{q^n}c \quad ,
\end{aligned} \tag{2}
$$

which is equivalent to (1). The argument can be iterated to rank any strategy which differ from each other by the exchange of the order of two areas. Since any permutation can be generated in this way, the result follows. □

Lemma 2

Proof. If the OCGs never see each others, they assume that the probability that the areas that they are not exploiting is $\frac{1}{1+\eta}$. Consider OCG j who lastly visited area m. For j, $q_j^m = 1$ while $q_j^{m'} = \frac{1}{1+\eta}$ for all $m' \neq m \in \{1, \ldots, M\}$. Given that the probability to find an area empty is equal for all areas. OCG j would never deviate from visiting m by visiting $m' > m$. Hence, j would only deviate to an area $m' \leq m$ where $m' = m$ only if $m = 1$ (i.e., the OCG visiting area 1 has no incentive to deviate). Using equation (2), we can show that OCG j will never deviate from m to m' if and only if

$$
u_{m'} - \frac{1}{\frac{1}{1+\eta}}c < u_m - c, \; \forall m' < m.
$$

When $\eta > \frac{u_1 - u_M}{c}$, the above inequality holds true for all $m' < m$. □

Proposition 1 and 2

We proceed by firstly showing OCGs equilibrium behavior and then derive level of concentration and frequencies of violence. We firstly derive OCGs' equilibrium behavior for $\underline{\eta} < \eta < \overline{\eta}$. We summarize these results in the following statement and then prove the results subsequently.

Lemma A.1. *Let i be the last OCG exploiting area* 1*, j the last OCG exploiting area* 2*, and ℓ the last OCG exploiting area* 3*. If* $\max\{\frac{u_1-u_2}{c}; \frac{u_2-u_3}{c}\} < \eta < \frac{u_1-u_3}{c}$*, then each OCG adopts the following strategy profile equilibrium:*

1. *OCG i plays $s_{1\to3\to2}$.*
2. *OCG j plays $s_{2\to1\to3}$ (i.e., always returns to area* 2*).*
3. *There is a threshold τ', such that if $t \geq \tau'$, ℓ plays $s_{1\to3\to2}$, while if $t < \tau'$, ℓ plays $s_{3\to1\to2}$ (i.e., returns to area* 3*).* □

Proof. Consider the behavior of j. She has four possible deviation paths: $s_{1\to2\to3}$, $s_{1\to3\to2}$, $s_{3\to2\to1}$ and $s_{3\to1\to2}$. Note that $q_j^3 = q_j^1$ while $q_j^2 = 1$ if the other two OCGs behave as in equilibrium. Given these probabilities, and given $u_1 > u_3$, if j ever deviates, she will follow the profile $s_{1\to2\to3}$ by Lemma 1. Using Equation (2), we know that the deviation $s_{1\to2\to3}$ cannot be optimal if $\eta > \frac{u_1-u_2}{c}$.

Next, consider the behavior of i. She knows that if the other OCGs play as in equilibrium, then $q_i^1 > q_i^3$, and $u_1 - \frac{1}{q_i^1}c > u_3 - \frac{1}{q_i^3}c$. Hence, i always prefers $s_{\ldots,1\to3}$ to $s_{\ldots,3\to1}$. With a similar reasoning it must also be that $q_i^1 > q_i^2$, and hence, that $u_1 - \frac{1}{q_i^1}c > u_2 - \frac{1}{q_i^2}c$. Again, i always prefers $s_{\ldots,1\to2}$ to $s_{\ldots,2\to1}$. However, to sustain the equilibrium profile, it must also be that

$$u_3 - \frac{1}{q_i^3}c > u_2 - \frac{1}{q_i^2}c.$$

Following the previous proofs, $q_i^2 = \frac{1}{1+\eta}$, while $q_i^3 = 1$ since if area 1 is occupied, it mus be that area 3 is free. Hence, i prefers $s_{1\to3\to2}$ to $s_{1\to2\to3}$ given that $\eta > \frac{u_2-u_3}{c}$.

Third, consider the behavior of ℓ. Given that ℓ lastly occupied area 3 and that the others play in equilibrium, it cannot be that $q_\ell^2 > q_\ell^1$; hence, it must be that ℓ prefers $s_{\ldots,1\to2}$ to $s_{\ldots,2\to1}$. Since i plays in equilibrium, and ℓ lastly occupied area 3, the probability of finding area 3 occupied for ℓ is 0. Hence, if $\eta < \frac{u_1-u_3}{c}$, there exists a τ' such that if $t \geq \tau'$, ℓ will also prefer $s_{\ldots,1\to3}$ to $s_{\ldots,3\to1}$. Relying on Lemma 1, it could be that

$$\tau' = \frac{1}{1+\eta}\log\left(\frac{\frac{u_1-u_3}{c}+1}{\frac{u_1-u_3}{c}-\eta}\right).$$

Moreover, if ℓ finds area 1 occupied, she also deduces that area 3 is free, and hence, she prefers $s_{1\to3\to2}$ to $s_{1\to2\to3}$ given that $\eta > \frac{u_2-u_3}{c}$. □

In this second Lemma we show the results under $\eta < \underline{\eta}$.

Lemma A.2. *Let i be the last OCG exploiting area 1, j the last OCG exploiting area 2, and ℓ the last OCG exploiting area 3. If $\eta < \max\{\frac{u_1-u_2}{c}; \frac{u_2-u_3}{c}\}$, these are the optimal behaviors in equilibrium.*

1. *Optimal behavior of OCG i. If the last time i visited 2 the area was occupied, if $t \geq \tau_1^{2o,3}$, i plays $s_{1\to2\to3}$, while if $t < \tau_1^{2o,3}$, i plays $s_{1\to3\to2}$. If the last time i visited 2 the area was free, if $t \geq \tau_1^{2f,3}$, i plays $s_{1\to2\to3}$, while if $t < \tau_1^{2f,3}$, i plays $s_{1\to3\to2}$.*

2. *Optimal behavior of OCG j. If the last time j visited area 1 was occupied, $t \geq \tau_2^{1o,2}$, j plays $s_{1\to2\to3}$, while if $t < \tau_2^{1o,2}$, j plays $s_{2\to3\to1}$. If the last time j visited area 1 was free, $t \geq \tau_2^{1f,2}$, j plays $s_{1\to2\to3}$, while if $t < \tau_2^{1f,2}$, j plays $s_{2\to3\to1}$.*

3. *Optimal behavior of OCG ℓ.*

   (a) *If the last time ℓ visited 3 ℓ also visited 1 and 2, then if $t \geq \tau_3^{2o,3}$, then ℓ plays $s_{1\to2\to3}$, if $\tau_3^{1o,3} \leq t < \tau_3^{2o,3}$, then ℓ plays $s_{1\to3\to2}$, and if $t < \tau_3^{1o,3}$, then ℓ plays $s_{3\to1\to2}$.*

   (b) *If the last time ℓ visited area 3, ℓ visited area 2 but not 1, then if $t \geq \tau_3^{2o,3}$, then ℓ plays $s_{1\to2\to3}$, if $\tau_3^{1f,3} \leq t < \tau_3^{2o,3}$, then ℓ plays $s_{1\to3\to2}$, and if $t < \tau_3^{1f,3}$, then ℓ plays $s_{3\to1\to2}$.*

   (c) *If the last time ℓ visited area 3, ℓ did not visit area 2, but the last time that ℓ visited area 2, ℓ also visited area 1, then if $t \geq \tau_3^{2f,3}$, then ℓ plays $s_{1\to2\to3}$, if $\tau_3^{1o,3} \leq t < \tau_3^{2f,3}$, then ℓ plays $s_{1\to3\to2}$, and if $t < \tau_3^{1o,3}$, then ℓ plays $s_{3\to1\to2}$.*

   (d) *If the last time ℓ visited area 3, ℓ did not visit area 2, and if the last time ℓ visited area 2 ℓ did not visited area 1, then if $t \geq \max\{\tau_3^{2f,3}; \tau_3^{1f,3}; \tau_\ell^{1,2}\}$, then ℓ plays $s_{1\to2\to3}$, and if $t < \min\{\tau_3^{2f,3}; \tau_3^{1f,3}; \tau_\ell^{1,2}\}$, ℓ plays $s_{3\to2\to1}$. If $\min\{\tau_3^{2f,3}; \tau_3^{1f,3}; \tau_\ell^{1,2}\} < t < \max\{\tau_3^{2f,3}; \tau_3^{1f,3}; \tau_\ell^{1,2}\}$, ℓ plays according to the rank given by $u_m - \frac{1}{q_i^m}c$.* □

Proof. First of all, since OCGs are visiting all areas with the same frequency, we can consider $p^1 = p^3 = p^3 = p$. Consider OCG i. When she lastly visited area 1, area 1 was free, and hence, $z_i^1 = 0$,

$$\begin{aligned} q_i^1(t) &= p\left(1 - e^{-\frac{t-t_i^1}{1-p}}\right) + e^{-\frac{t-t_i^1}{1-p}} \\ &= p + e^{-\frac{t-t_i^1}{1-p}}(1-p) \end{aligned}$$

Given that $p^2 = p$ too, and that the last time that OCG i visited area 2 it must have been at a time further than t_i^1, it must be that $q_i^1(t) > q_i^2(t)$, and a similar reasoning applies for area 3. Hence, OCG i starts from area 1.

To show that when i prefers $s_{1\to 2\to 3}$ to $s_{1\to 3\to 2}$, consider the situation in which OCG i found area 1 occupied. In that case, OCG i considers whether to start looking from area 2 or area 3. Note that if i visited area 3, the area 3 was free (because of the OCGs' behavior in equilibrium). Hence,

$$q_i^3(t) = p + e^{-\frac{t-t_i^3}{1-p}}(1-p).$$

However, it might have been that area 2 was found occupied at time t_i^2, in that case it must have been that $t_i^2 \geq t_i^3$. If $t_i^2 = t_i^3$, then $z_i^2 = 1$ and

$$q_i^2(t) = p\left(1 - e^{-\frac{t-t_i^3}{1-p}}\right).$$

Given that $t_i^2 = t_i^3$, it must be that $q_i^2(t) < q_i^3(t)$, and hence, we can find $\tau_1^{2o,3}$ by solving the following inequality

$$u_2 - \frac{1}{p\left(1 - e^{-\frac{t-t_i^3}{1-p}}\right)}c > u_3 - \frac{1}{p + e^{-\frac{t-t_i^3}{1-p}}(1-p)}c.$$

If $t_i^2 > t_i^3$, then $z_i^2 = 0$ and

$$q_i^2(t) = p\left(1 - e^{-\frac{t-t_i^2}{1-p}}\right) + e^{-\frac{t-t_i^2}{1-p}}.$$

With a similar reasoning we find $\tau_1^{2f,3}$ by solving the following inequality

$$u_2 - \frac{1}{p\left(1 - e^{-\frac{t-t_i^2}{1-p}}\right) + e^{-\frac{t-t_i^2}{1-p}}}c > u_3 - \frac{1}{p + e^{-\frac{t-t_i^3}{1-p}}(1-p)}c.$$

Next, consider OCG j. Given that she lastly occupied area 2, with a similar reasoning as the one above for i, it must be that $q_j^2 > q_j^3$. Hence, j always prefers $s_{\dots,2\to 3}$ to $s_{\dots,3\to 2}$. Similarly to q_i^1, it must be that

$$q_j^2(t) = p + e^{-\frac{t-t_j^2}{1-p}}(1-p)$$

Moreover, it must be that area 1 was found occupied at a time $t_j^1 \geq t_j^2$. If $t_j^1 = t_j^2$, then it must be that area 1 was occupied, and hence, $z_j^1 = 1$, and

$$q_j^1(t) = p\left(1 - e^{-\frac{t-t_j^1}{1-p}}\right)$$

We can find $\tau_2^{1o,2}$ solving the following inequality:

$$u_1 - \frac{1}{p\left(1 - e^{-\frac{t-t_j^1}{1-p}}\right)} c > u_2 - \frac{1}{p + e^{-\frac{t-t_j^2}{1-p}}(1-p)} c$$

If $t_j^1 > t_j^2$, then it must be that area 1 was free, and hence, $z_j^1 = 0$, and

$$q_j^1(t) = p\left(1 - e^{-\frac{t-t_j^1}{1-p}}\right) + e^{-\frac{t-t_j^1}{1-p}}$$

We can find $\tau_2^{1f,2}$ solving the following inequality:

$$u_1 - \frac{1}{p\left(1 - e^{-\frac{t-t_j^1}{1-p}}\right) + e^{-\frac{t-t_j^1}{1-p}}} c > u_2 - \frac{1}{p + e^{-\frac{t-t_j^2}{1-p}}(1-p)} c$$

Lastly, consider the behavior of OCG ℓ. Given she lastly occupied area 3, she assumes that area 3 is free. Given t_ℓ^3, it must be that $z_\ell^3 = 0$ and that $t_\ell^1 \geq t_\ell^2 \geq t_\ell^3$, and hence, we have to distinguish between 3 cases.

1. In the first, OCG ℓ visited all areas the last time she occupied 3. Hence, $t_\ell^1 = t_\ell^2 = t_\ell^3$ and $z_\ell^1 = z_\ell^2 = 1$.
2. In the second, OCG ℓ started from area 2 at time t_ℓ^3, Hence, it must be that $t_\ell^1 > t_\ell^2 = t_\ell^3$, $z_\ell^1 = 1$ and $z_\ell^2 = 0$
3. In the third, OCG ℓ started from area 3 at time t_ℓ^3. Hence, it mus be that $t_\ell^1 \geq t_\ell^2 > t_\ell^3$. First, we consider the case $t_\ell^1 = t_\ell^2 > t_\ell^3$ where $z_\ell^1 = 1$ and $z_\ell^2 = 0$.
4. In the fourth case, $t_\ell^1 > t_\ell^2 > t_\ell^3$, and hence, $z_\ell^1 = z_\ell^2 = z_\ell^3 = 0$.

Consider case 1). In this scenario,

$$q_\ell^1(t) = p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right)$$

$$q_\ell^2(t) = p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right).$$

Given the two above equations it must be that ℓ prefers $s_{\ldots,1\to2}$ to $s_{\ldots,2\to1}$. Moreover, we can calculate $\tau_3^{1o,3}$ such that ℓ prefers $s_{\ldots,1\to3}$ to $s_{\ldots,3\to1}$, and $\tau_3^{2o,3}$ such that ℓ prefers $s_{\ldots,2\to3}$ to $s_{\ldots,3\to2}$ solving the following inequalities

$$u_1 - \frac{1}{p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right)} c > u_3 - c.$$

$$u_2 - \frac{1}{p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right)}c > u_3 - c.$$

Note that since $u_1 > u_2$, it must be that $\tau_3^{1o,3} < \tau_3^{2o,3}$. Hence, if $t \geq \tau_3^{2o,3}$, then ℓ plays $s_{1\to 2\to 3}$, if $\tau_3^{1o,3} \leq t < \tau_3^{2o,3}$, then ℓ plays $s_{1\to 3\to 2}$, and if $t < \tau_3^{1o,3}$, then ℓ plays $s_{3\to 1\to 2}$.

In case 2),

$$q_\ell^1(t) = p\left(1 - e^{-\frac{t-t_\ell^1}{1-p}}\right) + e^{-\frac{t-t_\ell^1}{1-p}}$$

$$q_\ell^2(t) = p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right).$$

In this case, ℓ prefers $s_{\dots,2\to 3}$ to $s_{\dots,3\to 2}$ if $t \geq \tau_3^{2f,3}$. Moreover, we can calculate $\tau_3^{1f,3}$ such that

$$u_1 - \frac{1}{p\left(1 - e^{-\frac{t-t_\ell^1}{1-p}}\right) + e^{-\frac{t-t_\ell^1}{1-p}}}c > u_3 - c.$$

OCG ℓ prefers $s_{\dots,1\to 2}$ to $s_{\dots,2\to 1}$ if and only if

$$u_1 - \frac{1}{p\left(1 - e^{-\frac{t-t_\ell^1}{1-p}}\right) + e^{-\frac{t-t_\ell^1}{1-p}}}c > u_2 - \frac{1}{p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right)}c.$$

Given that $t_\ell^1 > t_\ell^2$, then it must be that $p\left(1 - e^{-\frac{t-t_\ell^1}{1-p}}\right) > p\left(1 - e^{-\frac{t-t_\ell^3}{1-p}}\right)$, and hence, that $\frac{1}{p\left(1-e^{-\frac{t-t_\ell^1}{1-p}}\right)+e^{-\frac{t-t_\ell^1}{1-p}}} < \frac{1}{p\left(1-e^{-\frac{t-t_\ell^3}{1-p}}\right)}$. Given that $u_1 > u_2$, we conclude that ℓ always prefers $s_{\dots,1\to 2}$ to $s_{\dots,2\to 1}$ in this case.

Similar to case 1), it must be that $\tau_3^{1f,3} < \tau_3^{2o,3}$. Hence, if $t \geq \tau_3^{2o,3}$, then ℓ plays $s_{1\to 2\to 3}$, if $\tau_3^{1f,3} \leq t < \tau_3^{2o,3}$, then ℓ plays $s_{1\to 3\to 2}$, and if $t < \tau_3^{1f,3}$, then ℓ plays $s_{3\to 1\to 2}$.

In case 3),

$$q_\ell^1(t) = p\left(1 - e^{-\frac{t-t_\ell^2}{1-p}}\right)$$

$$q_\ell^2(t) = p\left(1 - e^{-\frac{t-t_\ell^2}{1-p}}\right) + e^{-\frac{t-t_\ell^2}{1-p}}.$$

Note that ℓ must again prefer $s_{\dots,1\to2}$ to $s_{\dots,2\to1}$ since

$$u_1 - \frac{1}{p\left(1-e^{-\frac{t-t_\ell^2}{1-p}}\right)}c > u_2 - \frac{1}{p\left(1-e^{-\frac{t-t_\ell^2}{1-p}}\right)+e^{-\frac{t-t_\ell^2}{1-p}}}c,\ \forall t>0.$$

We already know $\tau_3^{1o,3}$, and we calculate $\tau_3^{2f,3}$ solving the following inequality

$$u_2 - \frac{1}{p\left(1-e^{-\frac{t-t_\ell^2}{1-p}}\right)+e^{-\frac{t-t_\ell^2}{1-p}}}c > u_3 - c.$$

Given that ℓ prefers $s_{\dots,1\to2}$ to $s_{\dots,2\to1}$, it must be that $\tau_3^{1o,3} < \tau_3^{2f,3}$. Hence, if $t \geq \tau_3^{2f,3}$, then ℓ plays $s_{1\to2\to3}$, if $\tau_3^{1o,3} \leq t < \tau_3^{2f,3}$, then ℓ plays $s_{1\to3\to2}$, and if $t < \tau_3^{1o,3}$, then ℓ plays $s_{3\to1\to2}$.

Lastly, consider case 4), where

$$q_\ell^1(t) = p\left(1-e^{-\frac{t-t_\ell^1}{1-p}}\right)+e^{-\frac{t-t_\ell^1}{1-p}}$$

$$q_\ell^2(t) = p\left(1-e^{-\frac{t-t_\ell^2}{1-p}}\right)+e^{-\frac{t-t_\ell^2}{1-p}}.$$

We can calculate $\tau_\ell^{1,2}$ such that

$$u_1 - \frac{1}{p\left(1-e^{-\frac{t-t_\ell^1}{1-p}}\right)+e^{-\frac{t-t_\ell^1}{1-p}}}c > u_2 - \frac{1}{p\left(1-e^{-\frac{t-t_\ell^2}{1-p}}\right)+e^{-\frac{t-t_\ell^2}{1-p}}}c.$$

Given the thresholds $\tau_3^{2f,3}$, $\tau_3^{1f,3}$ and $\tau_\ell^{1,2}$ we can calculate the optimal behavior of ℓ i the third case. Surely, if $t \geq \max\{\tau_3^{2f,3};\tau_3^{1f,3};\tau_\ell^{1,2}\}$, then ℓ plays $s_{1\to2\to3}$, and if $t < \min\{\tau_3^{2f,3};\tau_3^{1f,3};\tau_\ell^{1,2}\}$, ℓ plays $s_{3\to2\to1}$. If $\min\{\tau_3^{2f,3};\tau_3^{1f,3};\tau_\ell^{1,2}\} < t < \max\{\tau_3^{2f,3};\tau_3^{1f,3};\tau_\ell^{1,2}\}$, ℓ plays according to the rank in Lemma 1.

To conclude the proof, consider a situation in which $p^1 \neq p^2$. In this situation, it must be that area 1 is visited more frequently than area 2. However, given OCGs' behavior, it must be that whenever a OCG visit 1, and 1 is occupied, then the OCG visit 2. The same for area 3 when 2 is occupied. Hence, it cannot be in a steady state that $p^1 \neq p^2 \neq p^3$. $\square$

We now proceed with the proof of Proposition 1 and 2.

Proof of Proposition 1. First, consider $\overline{O}(\eta)$. From Lemma 2 and Lemma A.1 and A.2, we can deduce that whenever OCGs go out of their homes, they land in an area. Since they go out from their areas with frequency $\frac{\eta}{1+\eta}$, then the average concentration level is $\frac{\eta}{1+\eta}$ too.

Now consider $\eta > \overline{\eta}$. In this case, by Lemma 2, we know that OCGs specialize, and hence, given that they go out of their homes with frequency $\frac{\eta}{1+\eta}$, each area is occupied with frequency $\frac{\eta}{1+\eta}$.

If $\underline{\eta} < \eta < \overline{\eta}$, by Lemma A.1, we know that OCG j specializes in area 2. As before, the concentration level of area 2 must be $\frac{\eta}{1+\eta}$. We know that OCG i always starts from area 1 when going out, and that OCG ℓ occasionally goes to area 1. When ℓ goes to area 1, sometimes ℓ meets i, and sometimes i is at home. Hence, it must be that the concentration level of area 1 is bigger than $\frac{\eta}{1+\eta}$. Consider area 3. When i goes to 3 is must be that i and ℓ meet in 1, even though ℓ sometimes goes to 3 without going to 1. Since ℓ does not always go to 3 as the first area, and since when i goes there, she goes there after going to 1 it must be that $O_3(\eta) < \frac{\eta}{1+\eta}$.

The argument for $\eta < \underline{\eta}$ is similar to the above one, and hence, we omit it. □

Proof. Proof of Proposition 2 We begin the proof by noticing that $V_3(\eta) = 0$ follows directly from the previous propositions, as $\forall\eta$ OCGs go to 3 after having found other areas occupied, or because they specialize on it (see Lemma 2). With a similar reasoning, we also deduce $V_1(\eta)$ and $V_2(\eta)$ for $\eta > \overline{\eta}$ (again, see Lemma 2).

Now consider $\underline{\eta} < \eta < \overline{\eta}$. Again, only j visits area 2, and hence, it must be that $V_2(\eta) = 0$ for $\underline{\eta} < \eta < \overline{\eta}$. Now consider $V_1(\eta)$. Violent confrontations happen when i and ℓ visit the area at the same time. Since i visits such area at rate $\frac{\eta}{1+\eta}$ while ℓ visits that area at rate $\alpha_1^\ell \frac{\eta}{1+\eta}$, and since this two probabilities are independent, the probability of i and ℓ meeting in area 1 is the product of the two previous probabilities, i.e, $\alpha_1^\ell \left(\frac{\eta}{1+\eta}\right)^2$.

Lastly, consider $\underline{\eta} < \eta < \overline{\eta}$. Let us start from $V_1(\eta)$: a violent confrontation in this area happens when i and ℓ meet or when j and i meet in that area with a similar reasoning to the one above, it must be that for $\underline{\eta} < \eta < \overline{\eta}$, $V_1(\eta)$ is the sum of the joint probabilities of OCGs visiting 1. Hence, the value in the corollary. A similar reasoning applies for $V_2(\eta)$. □

Proof. Proof of Proposition 3 We start by showing the result for $\eta > \overline{\eta}$. If $\eta > \overline{\eta}$, then we know by Lemma 2 that given an OCG i who last exploited area m, then i's strategy will always be $s_{m,\ldots}$. Hence, we can conclude that for $\eta > \overline{\eta}$, $R_m(\eta) = \frac{\eta}{1+\eta}$ for all $m \in \{1, 2, 3\}$, where $\frac{\eta}{1+\eta}$ is the frequency with which OCGs go out of their turf.

When $\underline{\eta} < \eta < \overline{\eta}$, even we don't know with precision $R_1(\eta)$ and $R_3(\eta)$, we know for a fact that $R_2(\eta)$ must be bigger than the other two since by Lemma A.1, given an OCG i who last exploited area 2, then i's strategy will be $s_{2,\ldots}$ while for the other OCGs will never try to exploit area 2 since they would first try to exploit area 1 and then 3, again, by Lemma A.1.

If $\eta < \underline{\eta}$, then by Lemma A.2, we know that all OCGs can find the right belief to try to exploit area 1 first. In particular, we know that given an OCG i who lastly exploited area 1, then the probability that i's strategy will be $s_{1,\dots}$ is 1. However, the same cannot be said for the probability that an OCG j who last visited resource 2 (3) that j's strategy will be $s_{2,\dots}$ ($s_{3,\dots}$). Hence, whatever the relation between $R_2(\eta)$ and $R_3(\eta)$ will be, we can conclude that $R_1(\eta) > \max\{R_2(\eta); R_3(\eta)\}$. Moreover, Thanks to Lemma A.2, we know that it cannot be that $R_2(\eta) < R_3(\eta)$ since the last OCG exploiting area 2 might start from 1 the next time they will go out of their turf, but will never start from 3, while the last OCG exploiting area 3 might start from all of the three areas the next time they go out of their turf. $\square$

Corollary 1 and 2

Proof of Corollary 1. Let us start with $O_1(\eta)$. Area 1 is occupied occasionally by i or ℓ. More precisely, the probability of that area being occupied is the sum of the following three probabilities:

1. Probability that i goes to 1 and finds it free: $\frac{\eta}{1+\eta}\left((1-\alpha_1^\ell) * \frac{\eta}{1+\eta} + \frac{1}{1+\eta}\right)$.
2. Probability that ℓ goes to 1 and finds it free: $\alpha_1^\ell * \frac{\eta}{(1+\eta)^2}$.
3. Probability that ℓ and i meet in area 1: $\alpha_1^\ell * \left(\frac{\eta}{1+\eta}\right)^2$.

Hence,

$$O_1(\eta) = \frac{\eta}{1+\eta}\left((1-\alpha_1^\ell) * \frac{\eta}{1+\eta} + \frac{1}{1+\eta}\right) + \alpha_1^\ell * \frac{\eta}{(1+\eta)^2} + \alpha_1^\ell * \left(\frac{\eta}{1+\eta}\right)^2 \quad (3)$$

Now consider $O_1(\eta)$ when $\eta < \underline{\eta}$. Similarly to the previous case, we can write $O_1(\eta)$ as a sum of probabilities that OCGs visit that area alone and when they meet in that area. Given that for $\eta < \underline{\eta}$, also j visits area 1, it must be that $O_1(\eta)$ for this range of values of η is the right hand side of Equation (3) plus a strictly positive term. Hence, when $\eta \to \underline{\eta}$, from the left, $O_1(\eta)$ must be strictly bigger than $O_1(\eta)$ when $\eta \to \underline{\eta}$, from the right.

With a similar intuition, we can show that the limit from the left of $O_1(\eta)$ as $\eta \to \overline{\eta}$ is bigger than the one from the right. Indeed, As shown in Proposition 1, $O_1(\eta) = \frac{\eta}{1+\eta}$ if $\eta > \overline{\eta}$. Comparing the right hand side of (3) with $\frac{\eta}{1+\eta}$, it is evident that for similar numbers, the former is strictly bigger.

The intuition for $O_3(\eta)$ is very similar. Indeed, $O_3(\eta) = \frac{\eta}{1+\eta}$ if $\eta > \overline{\eta}$. However, from Proposition 1 we know that $O_3(\eta) < \frac{\eta}{1+\eta}$ for all $\eta < \overline{\eta}$. Hence, it must be that the limit from the left of $O_3(\eta)$ for $\eta \to \overline{\eta}$ is lower than the one from the right. Next, consider the limits from the left and from the right for $\eta \to \underline{\eta}$. Since OCG ℓ sometimes starts from area 2 too (see Lemma A.2), it must be that α_3^ℓ is smaller for $\eta < \underline{\eta}$ than for $\eta > \underline{\eta}$. Moreover, the frequency of times that i and j visit 3 cannot be bigger for $\eta < \underline{\eta}$ than for $\eta > \underline{\eta}$ since, for Lemma A.2, we know that both these OCGs never start from 3. Hence, it must be that $O_3(\eta)$ is lower for all $\eta < \underline{\eta}$ than for all $\eta > \underline{\eta}$, and so it should behave in the limits. $\square$

Proof of Corollary 2. The proof for $V_2(\eta)$ follows directly from Proposition 2, as $V_2(\eta) > 0$ for $\eta < \underline{\eta}$ and $V_2(\eta) = 0$ for $\eta > \underline{\eta}$. Similarly, it is straightforward to show $\lim_{\eta \to \overline{\eta}^-} V_1(\eta) > \lim_{\eta \to \overline{\eta}^+} V_1(\eta)$ as by Proposition 2, $V_1(\eta) > 0$ for $\eta < \overline{\eta}$ but $V_1(\eta) = 0$ for $\eta > \overline{\eta}$. Again, for Proposition 2, $V_1(\eta)$ is strictly bigger for $\eta < \underline{\eta}$ than for $\underline{\eta} < \eta < \overline{\eta}$, and consequently, it must be that $\lim_{\eta \to \underline{\eta}^-} V_1(\eta) > \lim_{\eta \to \underline{\eta}^+} V_1(\eta)$. $\square$

B Additional results from simulations

In this section, we show the results computed from various simulations of our theoretical model with more than 3 OCGs and more than 3 areas. Figure 1 and Figure 2 depicts the results of the simulation with 10 ares and 10 OCGs for different values of η. In both pictures we depict occupation levels, violence levels, and average number of streaks for the 10 areas; the main difference between the two figures is that, in the second, we divide the areas in three clusters according to profitability (according to our theoretical model).

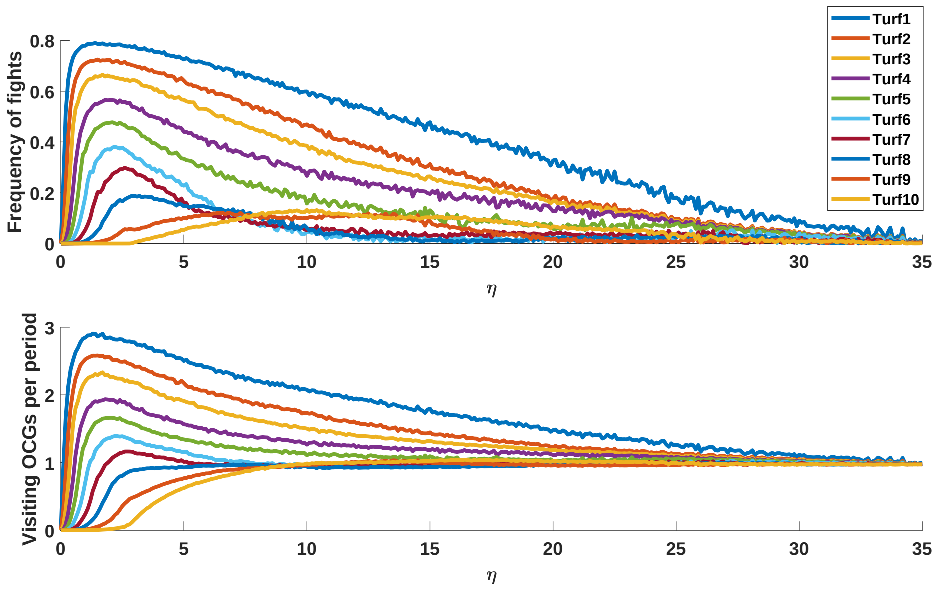

Figure 1: Violence and occupation levels, and average streak for different values of η when $m = 10$.

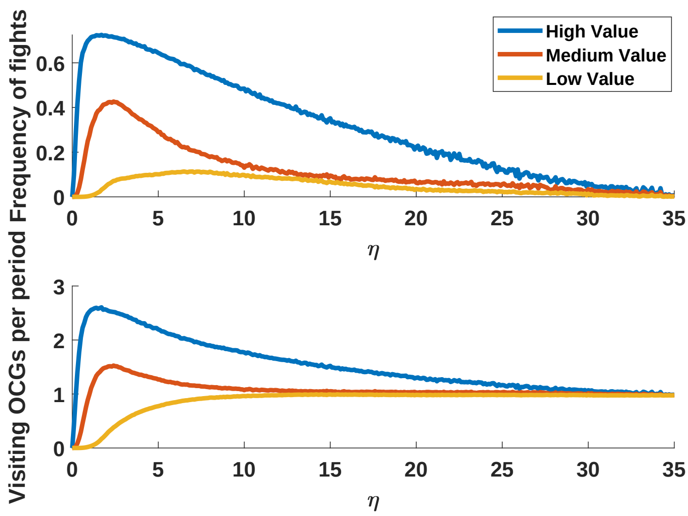

Figure 2: Violence and occupation levels, and average streak for different values of η when $m = 10$ and areas are aggregated according to profitability.